The Loss in Production Facilities as a Result of Exogenous Financial Shock

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1 Introduction

The purpose of this paper is to analyze the effects that an exogenous financial shock has on capital profitability. In particular, it will be assumed that the firm can choose only among a limited number of production functions, which are mutually exclusive and exploit capital at different intensities.

The analysis is microeconomic in nature and will be carried out by first consederating the deterministic case and then the probabilistic case. After stating the postulates and developing the model, an attempt will be made to answer the following question:

How large is the loss of profitability in a deterministic environment?

And further, given the stochastic case:

How much do the firm's expectations affect the profitability of capital?

The model assumes an initial situation of indifference among different plants and that the firm has already decided how to exploit capital at the time of the exogenous shock. Consequently, the cost of capital acquisition will not be taken into account, since it has already been incurred and is the same for each plant.

2 Assumptions

Assume that the firm produces only one type of output, using only capital as input, and that the selling price is normalized to 1. In addition, the firm chooses the type of production from 3 available, which are mutually exclusive. Each production method is described by a production function. Thus, the profit maximization problem is:

$$\max_{f_i,K}(\pi) = f_i - p \cdot K \tag{1}$$

Such that:

$$f_i \in \{f_1, f_2, f_3\} \tag{2}$$

Where π denotes precisely the profit, K the capital and p the cost of capital, taken as constant. Each production function has the following form:

$$f_i(K) = \sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{i,t} \cdot K \tag{3}$$

In (3) $\phi_{i,t}$ represents the return (output) on capital of function i for each period, discounted by $\frac{1}{1+r}$, where r is the market interest rate. The firm thus faces two costs, the cost of acquiring capital, and the opportunity cost of tying up capital in a given facility.

Assume further that the firm at time $t = 0$ is indifferent with respect to each mode of production. Given then capital K^*:

$$\pi(f_i, K^*) = \pi(f_j, K^*) \tag{4}$$

Substituting:

$$\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{i,t} = \sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{j,t} \tag{5}$$

Now, what distinguishes each production function is the series $\{\phi_t\}_1^3$, which can take infinite values. Assume that:

$$\phi_{1,t} < \phi_{1,t+1} \tag{6}$$

$$\phi_{2,t} > \phi_{2,t+1} \tag{7}$$

$$\phi_{3,t} = \phi_{3,t+1} \tag{8}$$

In other words, capital has decreasing, increasing, and constant returns, respectively.

Assume further:

$$t \in [0, 10] \tag{9}$$

And that the capital cannot be demobilized before time T.

To make the analysis clearer, approximate (3) to the continuous case:

$$f_i(K) = K \cdot \int_0^{10} e^{-rt} \cdot \phi_{i,t} \; dt \tag{10}$$

(6), (7) and (8) can now be explicitly described as follows:

$$\phi_{1,t} = -\alpha \cdot t + b \tag{11}$$

$$\phi_{2,t} = \gamma \cdot t \tag{12}$$

$$\phi_{3,t} = \delta \tag{13}$$

With $\alpha, \gamma, \delta > 0$. Given (9), and normalizing α to 1, (11) can be simplified as follows:

$$\phi_{1,t} = 10 - t \tag{14}$$

Thus, the production functions are:

$$f_1 = \int_0^{10} e^{-rt} \cdot (10 - t)) \; dt \tag{15}$$

$$f_2 = \int_0^{10} e^{-rt} \cdot \gamma \cdot t \; dt \tag{16}$$

$$f_3 = \int_0^{10} e^{-rt} \cdot \delta \; dt \tag{17}$$

K^* was omitted as equal for each f_i. Resolving the integrals:

$$f_1 = \frac{e^{-10r} + 10r - 1}{r^2} \tag{18}$$

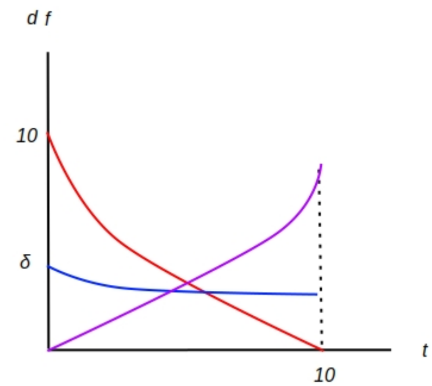

Figure 1: Productivity of capital in function of time

$$f_2 = \gamma \cdot \frac{1 - e^{-10r}(10r + 1)}{r^2} \tag{19}$$

$$f_3 = \delta \cdot \frac{1 - e^{-10r}}{r} \tag{20}$$

In Figure 1, it is possible to visualize the return on capital as a function of time; in red we have the argument of (15), in purple of (16) and in blue of (17).

3 Monetary Shock in a Totally Deterministic Environment

Assume that the firm has no information about financial market trends, that the benchmark interest rate is the government bond rate, and that the central bank decides to pursue a restrictive monetary policy from $t = \epsilon$, with ϵ subsiciently small [1]. In our model, this implies an increasing change in parameter r. As a result, the firm is no longer indifferent to different modes of production, but will face a larger or smaller loss, depending on the plant used.

[1]This simplifies the analysis, since the change in output does not take into account the capital already tapped from time 0 to *epsilon*

We measure the change in productivity by calculating the derivative of (18), (19) and (20) with respect to r:

$$\frac{df_1}{dr} = -\frac{e^{-10r}((10r-2)\cdot e^{10r} + 10r + 2)}{r^3} \tag{21}$$

$$\frac{df_2}{dr} = -\gamma \cdot \frac{2e^{-10r}(e^{10r} - 50r^2 - 10r - 1)}{r^3} \tag{22}$$

$$\frac{df_3}{dr} = \delta\left(\cdot\frac{10e^{-10r}}{r} - \frac{1-e^{-10r}}{r^2}\right) \tag{23}$$

(21), (22) and (23) are negative, indicating a change in yield moving in the opposite direction of the interest rate. To make the analysis clearer, let us assume an interest rate at time $t = 0$ equal to $r = 0.025$. Substituting in (18), (19) and (20) we find that the output at time 0 is equal to 48 and the parameters γ and δ are equal to 1.2 and 5.5, respectively. Finally, substituting for (21), (22) and (23) and calculating the percent change from an increase in r of 50, we find

$$\Delta\% f_1 \approx -16\% \tag{24}$$

$$\Delta\% f_2 \approx -19\% \tag{25}$$

$$\Delta\% f_3 \approx -8\% \tag{26}$$

As (24), (25) and (26) show, the largest loss occurs with f_2 and the smallest with f_3.

What we have seen is also true in the case of an expansionary monetary policy, for example, a change in r decreasing by 50%; in this case, the percentages are the same as above, but with a positive sign.

We can therefore draw the following conclusion:

P1: *Given a situation of total uncertainty about monetary policy decisions, capital immobilization in plants with constant (increasing) returns turns out to be the most precautionary (risky) choice.*

What we have seen thus does not affect the firm's initial choice, since its propensity to RISK and/or its expectations are not taken into account.

4 Financial Shock in a Stochastic Environment

Suppose the firm possesses information such that it assumes an upward change in the interest rate of 50% with a probability of p. Consequently, given the same capital productivity in each plant, the firm will choose the mode of production with smaller expected loss. The expected change in profit is equal to:

$$\Delta\pi_i^e = f_i^+ \cdot p + (1-p) \cdot f_i \tag{27}$$

In (27) f_i^+ represents the production in the case of a change in r and f_i the production in the case of constant r.

Substituting to the values found in the previous section, we obtain:

$$\Delta\pi_2^e = -p \cdot 8 \tag{28}$$

$$\Delta\pi_3^e = -p \cdot 9 \tag{29}$$

$$\Delta\pi_3^e = -p \cdot 4 \tag{30}$$

In each case, the expected loss is mitigated by the probability p, benefiting the riskier choice. For example, a probability of 0.2 returns an expected plant loss f_2 of 1.8, or 7.2 fewer units of output than in the deterministic environment [2]. The same holds in the case of a decrease in interest rates: the probability p in this case mitigates the increase in output.

It is not reasonable to assume that the central bank can pursue both restrictive and expansionary monetary policy with different probabilities. Thus, to make the model more plausible, assume that the reference interest rate is not the rate on government bonds, but the expected return on a similarly risky investment. In this case, the firm can assume both an upward variance with probability p and a downward variance with probability q [3]. Updating (27), we have the expected change in profit equal to:

[2] In each case, the loss is mitigated by $(1-p) \cdot 100$ percentage points less than in the deterministic case

[3] With $(1-p-q)$ we accordingly denote the probability that profit does not vary; this scenario is omitted because it does not affect the variance of profit

$$\Delta\pi_i^e = \Delta f_i^+ \cdot p + \Delta f_i^- \cdot q \tag{31}$$

Where f_i^- denotes the output in the case of a decrease in r.

Now, in the case $\Delta f_i^+ = \Delta f_i^-$ the result is the same as in the previous paragraph and the probability distribution plays no role. We then assume an increasing variance of 50%, as in the previous scenario, and a decreasing variance of 20%. Substituting in (31) for each production function, we get:

$$\Delta\pi_1^e = -p \cdot 8 + q \cdot 2 \tag{32}$$

$$\Delta\pi_3^e = -p \cdot 9 + q \cdot 1 \tag{33}$$

$$\Delta\pi_3^e = -p \cdot 4 + q \cdot 1.5 \tag{34}$$

As (32), (33) and (34) show, the expected profit depends on the probabilities p and q. We then equalize the change in profit to 0 to find the combination of probabilities in which the expected change is zero:

$$p \cdot = q \cdot \frac{1}{4} \tag{35}$$

$$p \cdot = q \cdot \frac{1}{9} \tag{36}$$

$$p \cdot = q \cdot \frac{3}{8} \tag{37}$$

Graphically, the different combinations of probabilities are shown in Figure 2, with the same color scheme as seen in Figure 1 [4]. The area above (below) the different curves indicates the combination of $\{p, q\}$ in which profit is positive (nevagative). As we can see, (37) has the steepest slope, suggesting a lower probability than an increase in profit. However, the curves extend for different areas, and to get a certain result we need to calculate the integral of (35), (36) and (37) [5]. The result is:

[4] The domain of the independent variable q was calculated by imposing for each function $p + q < 0$, so q lower than 0.8, 0.9 and 0.73, respectively

[5] For each function $\int_0^l (a \cdot q)\ dq$ was calculated, where a denotes the slope and l the upper bound of q

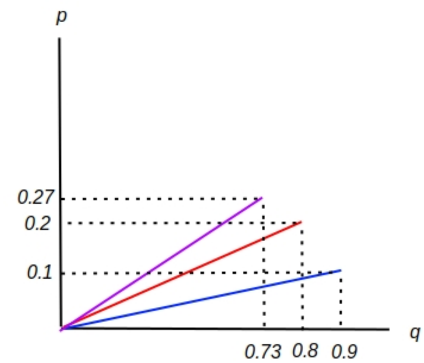

Figure 2: Probabilities that do not gives rise to change in profit

$$Pr(\Delta\pi_1^e < 0) = 0.08 \tag{38}$$

$$Pr(\Delta\pi_2^e < 0) = 0.05 \tag{39}$$

$$Pr(\Delta\pi_3^e < 0) = 0.1 \tag{40}$$

This confirms what was intuitively suggested: the production function f_3 has the highest probability of loss, while f_2 the lowest.

From what we have seen, we can draw the following conclusion:

P2: *Given the firm's expectations, there is a plant that guarantees an expected loss less than or equal to the other production methods.*

What is stated applies regardless of the different probabilities, but depends on the set of possible values that the interest rate can take in the future.

5 Conclusion

From the analysis performed we can conclude that in a situation of equal plant productivity and total uncertainty about the financial market trend, the choice of a particular production method does not *a priori* affect the profitability of capital.

In case the firm forms expectations about the interest rate trend, starting from a situation of indifference, there is one plant that weakly dominates over the others. In other words, for the same cost, there is a choice of capital use that guarantees a greater or equal return than its competitors.

One point to emphasize is the cost of production. The change in the interest rate negatively affects the cost of capital; in the analysis it was assumed that the cost of capital acquisition has already been incurred and that disinvestment is not possible before the end of the production cycle. The model is thus oriented toward the short run. In the long run, however, the firm changes its choices about the amount of inputs to be acquired, including other types of capital and labour.

Reference

The model developed in this paper is based on microeconomic theory. An excellent reference text is:

Walter Nicholson, Christopher M. Snyder, Microeconomic Theory: Basic Principles and Extensions; 12th Edition, 2016, Cengage Learning.