The Discrete Stochastic Gain-Loss in Production Facilities as a Result of an Exogenous Financial Shock

Gianmarco Corradini, MS

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1 Introduction

The following article is a follow-up to my previous paper *The Loss in Production Facilities as a Result of Exogenous Financial Shock* (2024). The first part remains basically the same; it introduces the model and draws initial conclusions in a deterministic environment. In the second part, the stochastic environment is introduced instead. It is an expansion of those seen earlier: more variability is allowed to the rate of interest and two states are taken into account.

The question to be answered is as follows:

How much does an expected multiple change in interest affect profitability?.

Even in this paper, the reference is the macroeconomic theory. Elements from game theory are also present. To make the discussion clearer, the model considers the standard deviation only indirectly. Also, the distribution function is of a discrete type.

Finally, the presence of multiple probabilities and states exponentially increased the amount of computation. It was therefore necessary to make use of the computer tool, particularly the Python language. A link from which the source code can be downloaded has been included in the references.

2 Assumptions

Assume that the firm produces only one type of output, using only capital as input, and that the selling price is normalized to 1. In addition, the firm chooses the type of production from 3 available, which are mutually exclusive. Each production method is described by a production function. Thus, the profit maximization problem is:

$$\max_{f_i, K}(\pi) = f_i - p \cdot K \tag{1}$$

Such that:

$$f_i \in \{f_1, f_2, f_3\} \tag{2}$$

Where π denotes precisely the profit, K the capital and p the cost of capital, taken as constant. Each production function has the following form:

$$f_i(K) = \sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{i,t} \cdot K \tag{3}$$

In (3) $\phi_{i,t}$ represents the return (output) on capital of function i for each period, discounted by $\frac{1}{1+r}$, where r is the market interest rate. The firm thus faces two costs, the cost of acquiring capital, and the opportunity cost of immobilizing capital in a given facility.

Assume further that the firm at time $t = 0$ is indifferent with respect to each mode of production. Given then capital K^*:

$$\pi(f_i, K^*) = \pi(f_j, K^*) \tag{4}$$

Substituting:

$$\sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{i,t} = \sum_{t=0}^{\infty}(\frac{1}{1+r})^t \cdot \phi_{j,t} \tag{5}$$

Now, what distinguishes each production function is the series $\{\phi_t\}_1^3$, which can take infinite values. Assume that:

$$\phi_{1,t} < \phi_{1,t+1} \tag{6}$$

$$\phi_{2,t} > \phi_{2,t+1} \tag{7}$$

$$\phi_{3,t} = \phi_{3,t+1} \tag{8}$$

In other words, the capital has a decreasing, increasing and constant return, respectively.

Assume further:

$$t \in [0, 10] \tag{9}$$

And that the capital cannot be demobilized before time T.

To make the analysis clearer, approximate (3) to the continuous case:

$$f_i(K) = K \cdot \int_0^{10} e^{-rt} \cdot \phi_{i,t} \, dt \tag{10}$$

(6), (7) and (8) can now be explicitly described as follows:

$$\phi_{1,t} = -\alpha \cdot t + b \tag{11}$$

$$\phi_{2,t} = \gamma \cdot t \tag{12}$$

$$\phi_{3,t} = \delta \tag{13}$$

With $\alpha, \gamma, \delta > 0$. Given (9), and normalizing α to 1, (11) can be simplified as follows:

$$\phi_{1,t} = 10 - t \tag{14}$$

Thus, the production functions are:

$$f_1 = \int_0^{10} e^{-rt} \cdot (10 - t)) \, dt \tag{15}$$

$$f_2 = \int_0^{10} e^{-rt} \cdot \gamma \cdot t \, dt \tag{16}$$

$$f_3 = \int_0^{10} e^{-rt} \cdot \delta \, dt \tag{17}$$

K^* was omitted as equal for each f_i. Resolving the integrals:

$$f_1 = \frac{e^{-10r} + 10r - 1}{r^2} \tag{18}$$

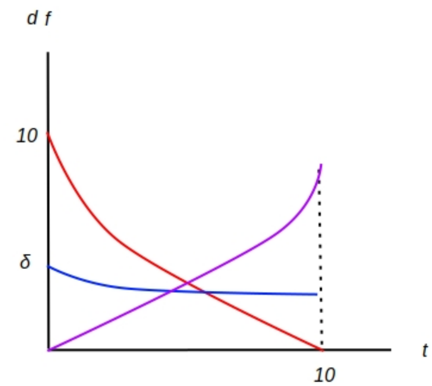

Figure 1: Productivity of capital in function of time

$$f_2 = \gamma \cdot \frac{1 - e^{-10r}(10r + 1)}{r^2} \tag{19}$$

$$f_3 = \delta \cdot \frac{1 - e^{-10r}}{r} \tag{20}$$

In Figure 1, it is possible to visualize the return on capital as a function of time; in red we have the argument of (15), in purple of (16) and in blue of (17). In contrast, with the same color scheme, (18), (19) and (20) are shown in Figure 2.

3 Expected Exogenous Financial Shock

Suppose $r = 0.025$ and the firm waits for two interest rate changes at different times sufficiently close to time 0 [1]. The probability distribution of r is the same at each instant and independent of previous changes:

$$Pr(\Delta_{\%} r = 25\%) = p \tag{21}$$

$$Pr(\Delta_{\%} r = 0) = (1 - p - q) \tag{22}$$

[1] This way we can ignore the amount of output already produced

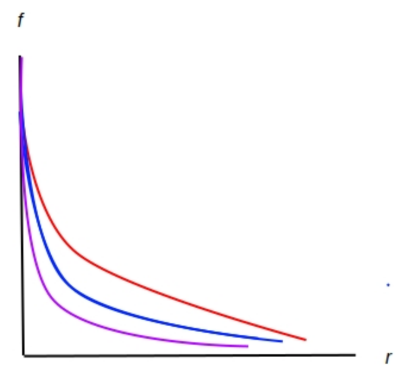

Figure 2: Productivity of capital in function of interest rate

$$Pr(\Delta_{\%}r = -10\%) = q \tag{23}$$

So we have 2 variations with 3 different probabilities, resulting in 2^3 possible outcomes, as described by the following table:

N	$f_i(r)$	Pr
1	$f_i(0.039)$	p^2
2	$f_i(0.031)$	$p \cdot (1-p-q)$
3	$f_i(0.028)$	$p \cdot q$
4	$f_i(0.031)$	$(1-p-q) \cdot p$
5	$f_i(0.025)$	$(1-p-q)^2$
6	$f_i(0.023)$	$(1-p-q) \cdot q$
7	$f_i(0.028)$	$q \cdot p$
8	$f_i(0.023)$	$q \cdot (1-p-q)$
9	$f_i(0.02)$	q^2

Substituting $r = 0.025$ into (18), we find that the output at $t = 0$ is equal to 46; equalizing to (19) and (20) we find γ and δ equal to 1.1 and 5.2, respectively.

We then calculate the output for each production function:

N	$f_1(r)$	$f_2(r)$	$f_3(r)$
1	44.08	44.07	43.11
2	45.17	44.25	44.73
3	45.62	45.15	45.4
4	45.17	44.25	44.73
5	46.08	46.08	46.08
6	46.45	46.84	46.64
7	45.62	45.15	45.4
8	46.45	46.84	46.64
9	46.79	47.54	47.15

The expected output of each plant is found by summing each output by its relative probability:

$$E(f_i) = p^2 \cdot f_i(0.039) + p \cdot (1-p-q) \cdot 2 \cdot f_i(0.031) + \tag{24}$$

$$\text{p} \cdot q \cdot 2 \cdot f_i(0.028) + (1-p-q)^2 \cdot f_i(0.025) + (1-p-q)q \cdot 2 \cdot f_i(0.023) + q^2 \cdot f_i(0.02)$$

Adjusting terms and substituting for each production function:

$$E(f_2) = p^2 \cdot (-0.18) + p \cdot (-1.82) + \tag{25}$$

$$\text{p} \cdot q \cdot (0.16) + q \cdot (0.74) + q^2 \cdot (-0.03) + 46.08$$

$$E(f_2) = p^2 \cdot (1.65) + p \cdot (-3.91) + \tag{26}$$

$$\text{p} \cdot q \cdot (-3.32) + q \cdot (1.52) + q^2 \cdot (-0.06) + 46.08$$

$$E(f_3) = p^2 \cdot (-0.27) + p \cdot (-4.14) + \tag{27}$$

$$\text{p} \cdot q \cdot (-1.02) + q \cdot (-0.32) + q^2 \cdot (-0.05) + 46.08$$

We then have the expected profit as a function of p and q, with domain $D = \{p, q \mid p + q < 0\}$. Let us now assume:

$$Pr(\Delta_{\%} r = 0) = \frac{1}{3} \tag{28}$$

The domain becomes $D = \{p, q | \ p = -q \cdot \frac{2}{3}\}$. Thus (25), (26) and (27) have different values for each value of p, which gives rise to a single value of

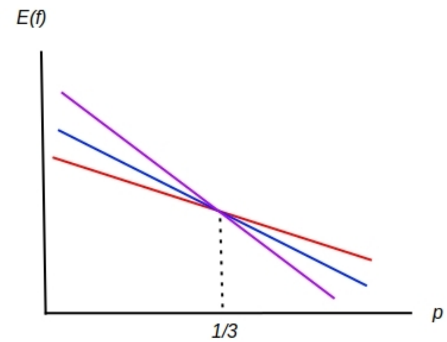

Figure 3: Expected production as a function of probability of an interest increase

q. In Figure 3 it is possible to see $E(f_i)$ as a function of p, with the same color scheme seen above. As can be seen, for p lower (greater) than $\frac{1}{3}$, the function f_2 (f_1) dominates over the others.

We can therefore draw the following conclusion:

P3: *Given the probability $P(\Delta r = 0) = \frac{1}{3}$, the constant (decreasing) exploitation of capital guarantees a higher expected return than the other modes of production for p lower (higher) than $\frac{1}{3}$.*

Note how the different probability distribution of r gives rise to different results. In particular, for a uniform distribution [2], the firm is indifferent with respect to each production function. In the case instead of a distribution with left (right) skewness, the firm prefers constant (decreasing) capital exploitation. Increasing capital exploitation, on the other hand, is weakly lower than the other production methods.

4 Conclusion

The analysis performed basically confirmed and reinforced what was seen in *The Loss in Production Facilities as a Result of Exogenous Financial Shock.*

[2] This implies that $p = q = \frac{1}{3}$

In particular, except in the case of a uniform probability distribution, it is possible to divide the set of all possible distributions into two subsets, where in each one production function dominates strongly over the others.

Also in the present work, the analysis was oriented to the short run. Indeed, the cost of capital acquisition and the possibility of demobilizing it before the end of the production cycle was not taken into account.

Last point concerns the company's appetite for risk. These preferences have not been described, and it should be noted that a specific utility function can substantially reverse what has just been seen.

Reference

The model developed in this paper is based on microeconomic theory. An excellent reference text is:

Walter Nicholson, Christopher M. Snyder, Microeconomic Theory: Basic Principles and Extensions; 12th Edition, 2016, Cengage Learning.

The source code used for the computations is in the Python language, with the extension .ipynb (Jupyter Notebook file), and available at the following link:

https://giacorradini.github.io/archive